

Statefinder Diagnostic for Born-Infeld Type Dark Energy Model

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Using a new method—statefinder diagnostic which can differ one dark energy model from the others, we investigate in this letter the dynamics of Born-Infeld(B-I) type dark energy model. The evolutive trajectory of B-I type dark energy with Mexican hat potential model with respect to e – *folding* time N is shown in the $r(s)$ diagram. When the parameter of noncanonical kinetic energy term $\eta \rightarrow 0$ or kinetic energy $\dot{\varphi}^2 \rightarrow 0$, B-I type dark energy(K-essence) model reduces to Quintessence model or Λ CDM model corresponding to the statefinder pair $\{r, s\}=\{1, 0\}$ respectively. As a result, the the evolutive trajectory of our model in the $r(s)$ diagram in Mexican hat potential is quite different from those of other dark energy models.

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With the increase of the modern precise cosmology[1-3], scientists believe more and more such a fact: the Universe is nearly flat to high precision, $\Omega_{total} = 0.99 \pm 0.03$, and is undergoing a accelerated expansion phase. To explain such a fact, an exoteric energy density which is an unclumped form of energy density pervading the Universe should be introduced. This unknown energy density called "dark energy" with negative pressure, contributes to about two thirds of the total energy density, and the density of clustered matter including cold dark matters plus baryons contributes to about one thirds of the total energy density. Perhaps the simplest explanation for these data is that the dark energy corresponds to a positive cosmological constant. However, there are two serious problems with the cosmological constant, namely the "fine-tuning" and the cosmic "coincidence". In the framework of quantum field theory, the expectation value of vacuum energy is 123 order of magnitude larger than the observed value of $10^{-47} GeV^4$. The absence of a fundamental mechanism which sets the cosmological constant zero or very small value is the first cosmological constant problem. The second problem as the cosmic coincidence, states that why are the energy densities of dark energy and dark matter nearly equal today? An alternative is a scalar field which has not yet reached its ground state.

It is well known that Hubble parameter $H(t) \equiv \frac{\dot{a}}{a}$ and deceleration parameter q_0 are very important quantities which can describe the geometrical properties of the Universe. However, both quantities can't provide enough evidence to differentiate the more accurate cosmological observational data and the more general models of dark energy. For this reason, Sahni et al[4] propose a new geometrical diagnostic pair $\{r, s\}$ for dark energy, which is called statefinder and can be expressed as follows.

$$r \equiv \frac{\ddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-\frac{1}{2})} \quad (1)$$

Obviously, this diagnostic is constructed from the $a(t)$ and its derivatives up to the third order. So, the statefinder probes the expansion dynamics of the universe through higher derivatives of the expansion factor. By far, many models[5] have been differentiated by this geometrical diagnostic method. Its important property is that $\{r, s\} = \{1, 0\}$ is a fixed point for the flat Λ CDM FRW cosmological model. Departure of a given DE model from this fixed point is a good way of establishing the "distance" of this model from flat Λ CDM. In this letter, we will investigate the evolutive trajectory of B-I type dark energy model in the $r-s$ diagram when the potential is taken as the Mexican hat potential, and show the "distance" between our model and Λ CDM model.

Born and Infeld firstly introduce a nonlinear electromagnetic field in 1934, and their original motivation is to resolve the singularity in classical electromagnetic dynamics[6]. So far, many authors have been studying the nonlinear B-I type string theory and cosmology[7]. The lagrangian density for a B-I type scalar field is

$$L_S = \frac{1}{\eta} \left[1 - \sqrt{1 - \eta g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu}} \right] \quad (2)$$

When parameter of noncanonical kinetic energy η tends to be zero, by Taylor expansion, Eq.(2) approximates to the lagrangian of linear scalar field. This means that B-I type dark energy model will reduce to Quintessence model if $\eta \rightarrow 0$.

$$\lim_{\eta \rightarrow 0} L_S = \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} \quad (3)$$

Now we consider the Lagrangian with a potential $u(\varphi)$ in spatially homogeneous scalar field, so, Eq.(2) becomes

$$L_{ph} = \frac{1}{\eta} \left[1 - \sqrt{1 - \eta \dot{\varphi}^2} \right] - u(\varphi) \quad (4)$$

From Eqs.(9) and (10), we know when kinetic energy $\dot{\varphi}^2 \rightarrow 0$, B-I type dark energy model reduces to Λ CDM model. In FRW space-time metric, Einstein equation $G_{\mu\nu} = K T_{\mu\nu}$ can be written as

$$H^2 = \frac{1}{3}(\rho_\varphi + \rho_m) \quad (5)$$

$$\dot{H} = -\frac{1}{2}(\rho_\varphi + p_\varphi + \rho_m) \quad (6)$$

$$\dot{\rho}_m + 3H\rho_m = 0 \quad (7)$$

$$\dot{\rho}_\varphi + 3H(\rho_\varphi + p_\varphi) = 0 \quad (8)$$

where ρ_m , ρ_φ and p_φ are the matter energy density, the effective energy density and effective pressure of the B-I type scalar field respectively, and we work in units $8\pi G = 1$. ρ_φ and p_φ can be expressed as follows

$$\rho_\varphi = T_0^0 = \frac{1}{\eta \sqrt{1 - \eta \dot{\varphi}^2}} - \frac{1}{\eta} + u(\varphi) \quad (9)$$

$$p_\varphi \equiv \omega_\varphi \rho_\varphi = -T_1^1 = -T_2^2 = -T_3^3 = \frac{1}{\eta} - \frac{\sqrt{1 - \eta \dot{\varphi}^2}}{\eta} - u(\varphi) \quad (10)$$

According to Eq.(1) and Eq.s(5-8), we can obtain the expression of statefinder pair and deceleration parameter q

$$r = 1 + \frac{9}{2} \Omega_\varphi \omega_\varphi (1 + \omega_\varphi) - \frac{3}{2} \Omega_\varphi \omega'_\varphi \quad (11)$$

$$s = 1 + \omega_\varphi - \frac{1}{3} \frac{\omega'_\varphi}{\omega_\varphi} \quad (12)$$

$$q = \frac{1}{2} (1 + 3\Omega_\varphi \omega_\varphi) \quad (13)$$

where Ω_φ and ω_φ are parameter of energy density and state equation respectively, and a prime sign denotes the derivative with respect to the "e-folding" time $N = \ln a$.

Mexican hat potential $u(\varphi) = \frac{\mu}{4}(\varphi^2 - \varepsilon^2)^2 + u_0$ with μ , ε and u_0 being constant, has been widely studied in symmetry breaking problem of unification theory. In this letter, we take this potential as the potential of B-I type scalar field.

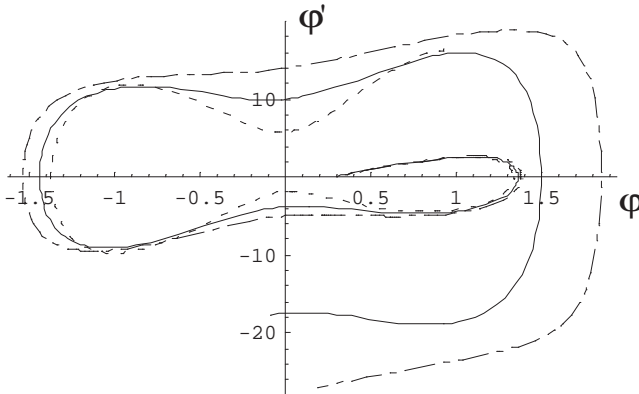


Fig.1 The attractive trajectories of B-I model with Mexican hat potential are stable spirals for different $\eta = 0.6, 1, 1.3$ respectively. We set $\mu = 0.5$, $\varepsilon = 0.95$, $u_0 = 1$.

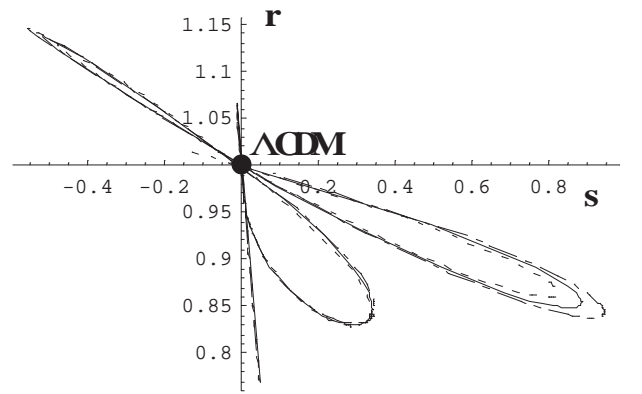


Fig.2 The evolutive trajectories of $r(s)$ for different $\eta = 1$ (real line), 0.6 (dot line), 1.3 (dot-dashed line) respectively.

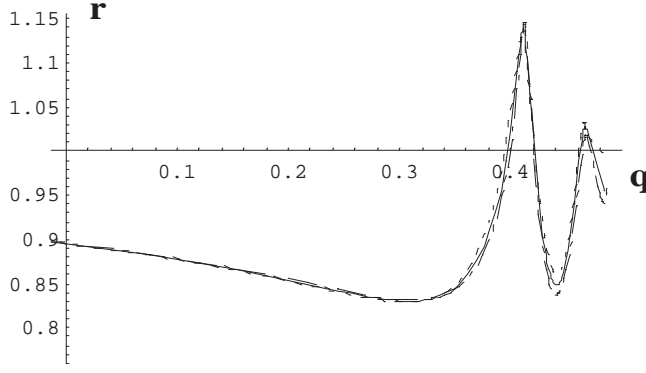


Fig.3 The evolutive trajectories of $r(q)$ for different $\eta = 1$ (real line), 0.6 (dot line), 1.3 (dot-dashed line) respectively.

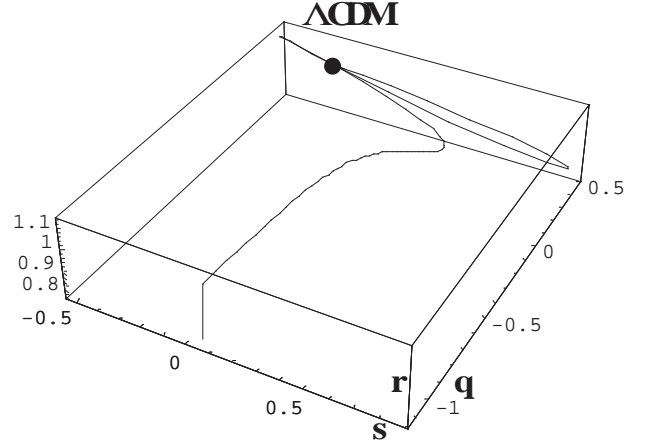


Fig.4 The 3D evolutive trajectories of $r - s - q$ for $\eta = 1$.

In summary, we investigate in this letter the dynamics of B-I type dark energy model by using a new geometric diagnostic method—statefinder pair $\{r, s\}$. We find that when $\eta \rightarrow 0$, B-I type dark energy (K-essence) model reduces to Quintessence model with canonical kinetic energy term. The phase portrait of $r - s$, $r - q$ and $r - s - q$ with respect to $e - folding$ time N are shown mathematically. Fig.1 shows that the B-I type dark energy model in Mexican hat potential admits a stable spiral attractor solution corresponding to the dark energy density $\Omega_\phi \sim 1$ and the equation of state parameter $\omega_\phi \sim -1$ meeting the current observations well. The new geometric quantities r and s possesses some interesting characters e.g. the evolutive trajectory of statefinder pair focuses on a fixed point $\{r, s\} = \{1, 0\}$ corresponding to Λ CDM model. Fig.2 shows that the evolutive trajectories of $r(s)$ will always pass the fixed point $\{r, s\} = \{1, 0\}$ of Λ CDM FRW cosmological model in the future when we set different value of $\eta = 0.6, 1, 1.3$ respectively. We can see easily the evolutive trajectories form series swirls before reaches the attractor, and are quite different from those of the other dark energy models whose swirl is none or only one. The phase portrait of $r - q$ and 3D parametric portrait of $r - s - q$ are shown in Fig.3 and Fig.4 respectively, where the parameter values are the same to those of Fig.1. As a result, the evolutive behavior of B-I type dark energy model in $r - s$ diagram will pass the fixed point corresponding to Λ CDM model and is quite different from those of other dark energy models.

Acknowledgements

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References

1. A. G. Riess, *Astron. J***116**, 1009(1998);
S. Perlmutter et al., *Astrophys. J***517**, 565(1999).
2. D. N. Spergel et al., *Astrophys. J. Suppl***148**, 175(2003).
3. P. de Bernardis et al., arXiv:astro-ph/0105296.
4. V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, *JETP Lett***77**, 201(2003).
5. B. R. Chang, H. Y. Liu, L. X. Xu and C. W. Zhang, *Chin. Phys. Lett.***24**, 2153(2007);
B. R. Chang, H. Y. Liu, L. X. Xu, C. W. Zhang and Y. L. Ping, *JCAP***0701**, 016(2007);
Gorini, A. Kamenshchik and U. Moschella, *Phys. Rev. D***67**, 063509(2003);
X. Zhang, *Phys. Lett. B***611**, 1(2005).
6. M. Born and Z. Infeld, *Proc. Roy. Soc. A***144**, 425(1934).
7. G. W. Gibbons and C. A. R. Herdeiro, *Phys. Rev. D***63**, 064006(2001);
H. P. de Oliveira, *J. Math. Phys.***36**, 2988(1995);
T. Taniuti, *Prog. Theor. Phys.(kyoto) Suppl***9**, 69(1958).